THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2078 (Term 2, 2023-24)
Honours Algebraic Structures
Course Outline

Outline

This course is an introduction to modern abstract algebra and the algebraic way of thinking in advanced mathematics. The course focuses on basic algebraic concepts which arise in various areas of advanced mathematics, and emphasizes on the underlying algebraic structures which are common to various concrete mathematical examples.

Topics include:

- Group Theory - examples of groups including permutation and dihedral groups, subgroups, the Theorem of Lagrange, group homomorphisms, normal subgroups and quotient groups.
- Ring Theory - examples of rings including the ring of integers and polynomial rings, integral domains, fields, ring homomorphisms, ideals and quotient rings.
- Field Theory - examples of field extensions and finite fields.

Prerequisites

Students taking this course should have taken MATH 1010/1018 *University Mathematics*, MATH 1050/1058 *Foundation of Modern Mathematics* and MATH 1030/1038 *Linear Algebra I* (or classes at equivalent level).

Class Information

- Instructor: CHAN Kwok Wai (Office: LSB 212; Email: `kwchan@math.cuhk.edu.hk`)
- Teaching Assistant:
  - Dr. NG Ming Ho (Office: LSB 223; Email: `mhng@math.cuhk.edu.hk`)
  - Mr. LAM Chin Hang Eddie (Office: LSB 222B; Email: `echlam@math.cuhk.edu.hk`)
- Lectures: Mon 1:30pm - 2:15pm at LSB LT4; Wed 4:30pm - 6:15pm at MMW 705
- Tutorials: Mon 2:30pm - 3:15pm at LSB LT4
- Webpage: `https://www.math.cuhk.edu.hk/course/2324/math2078`

Assessment

- 10%: Homework
- 30%: Midterm (12^{th} March 2024, Tuesday night)
- 60%: Final (TBA)

Suggested Texts

- Lecture notes available at the course webpage.
- J. A. Gallian, *Contemporary Abstract Algebra*, CMC Press, 10th edition (recommended).
- M. Artin, *Algebra*, Prentice Hall, 2nd edition.
- J. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley, 7th edition.
- P. Aluffi, *Algebra: Chapter 0*, Graduate Studies in Mathematics Vol. 104, American Mathematical Society.
- D. Dummit and R. Foote, *Abstract Algebra*, John Wiley and Sons, 3rd edition.

Lectures, Tutorials and Homeworks

Lectures: The lectures will focus mainly on the theoretical concepts and proofs, supplemented occasionally with some illustrative examples. As the lectures would only cover the most essential materials (at a rather fast pace), it would be very helpful if you have read (or at least skimmed through) the relevant chapters beforehand.

Tutorials: The tutorials will cover more concrete examples and computational aspects of the materials. The style is more informal and students are encouraged to discuss and work out some exercises together. All the materials (except otherwise stated) covered in lectures and tutorials will be covered in the midterm and final exam.

Homework assignments: Problem sets will usually be posted on Mondays and due on Thursday in the following week. Each problem set consists of two parts – the compulsory part and the optional part. Please only hand in your solutions of the compulsory part. But you are highly recommended to work out the optional part at home as well. The full mark for each homework is 1 point. There will be around 10 sets of homework assignments.

Tentative Schedule

- Week 1–6: Group theory
  - definition and basic examples of groups
  - cyclic groups, symmetric groups and dihedral groups
  - subgroups, cyclic subgroups and generating sets
  - cosets and the Theorem of Lagrange
  - normal subgroups and quotient groups
  - group homomorphisms and isomorphisms
- Week 7–12: Ring theory
  - definition and basic examples of rings
  - polynomials and polynomial rings
  - integral domains and fields; field of fractions
  - ring homomorphisms and isomorphisms

  - subrings and ideals
  - quotient rings
  - factorization of polynomials, Euclidean algorithm, PID
  - Gauss' Lemma and Eisenstein's Criterion
- Week 13: Field theory
  - basic theory of field extensions and finite fields